

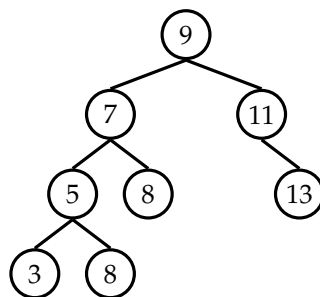
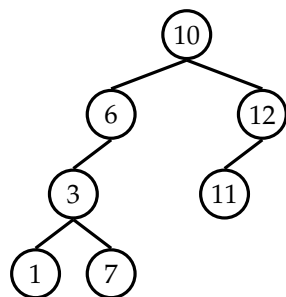
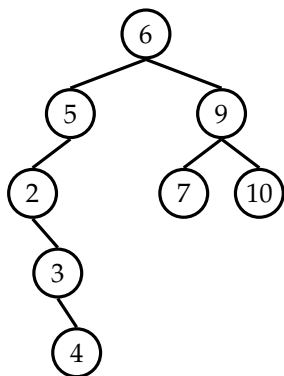
## 1 2-3-Forever

- 1.1 Draw what the 2-3 tree would look like after each step in inserting 18, 12, and 13.

Make sure to know how to convert each of these into left-leaning red-black trees!

## 2 Unbalanced Search Trees

- 2.1 Given the following binary trees, determine if each is a binary search tree, and whether the height of the tree is the same as the height of the optimal binary search tree containing the given elements.



- 2.2 Provide tight asymptotic runtime bounds in terms of  $N$ , the number of nodes in the tree, for the following operations and data structures.

Operations	BST	Red-black tree
<code>boolean contains(T o);</code>		
<code>void insert(T o);</code>		
<code>T get(int i);</code>		

### 3 Disjointed Sets

- 3.1 Suppose we have a `WeightedQuickUnionUF` disjoint set with path compression. Show the tree structure in the union-find algorithm as the following sequence of commands is executed.

(a) `connect(1, 2);`  
`connect(3, 4);`  
`connect(5, 6);`  
`connect(1, 6);`

(b) `find(6);`

- 3.2 Describe how to construct a `WeightedQuickUnionUF` tree of maximum height.